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Article

A Quanti-Qualitative Approach to Alexander's Harmony-Seeking Computations

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Abstract
Harmony-seeking computations, as proposed by Christopher Alexander, offer a way to tackle complexity. Smart, free agents, facing uncertainty, look for order in a context powered by fifteen attractors, or patterns. Harmony-seeking would then be a relatively guided path across those idealized patterns, towards wholeness and beauty. However, individuals acting to change the city must combine circumstances imposed by external and inner urban forces with personal interpretations of one or more of those patterns that could change all the time. Moreover, each action is intertwined with others, in an unpredictable outcome. This article explores the possibility of bringing together urban inner and outer forces and ingenious individuals' actions of city change by hypothesizing: (a) wholeness as a structural attribute defined as spatial centrality; (b) beauty as meaning attached to places, evolving either from historic accumulation or individual assignment; (c) order as every meaningful approximation between them; (d) a disaggregated description of the urban organism, based on multi-layered graphs, in which would be possible to record both morphological and territorial characteristics (form, transport, infrastructure) and semantic attributes (land uses, public image, remote associations, symbolic relationships); and (e) a set of spatial differentiation measures, mostly based on centrality, potentially able to depict wholeness (by measuring the effect of each component on all others) and beauty (by measuring urban robustness derived from any selected set of components). A multilayer graph-based approach to spatial differentiation algorithms provides a framework for the description, analysis, and performance evaluation of every component, as well as the whole system, both through quantitative and qualitative representation.

Keywords
graph representation; harmony-seeking; multilayer networks; spatial differentiation; urban planning and design; wholeness

Issue
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1. Introduction

Harmony-seeking (HS) computations is where Alexander (2009) takes on complexity, in his own terms: (a) there is order in nature (as well as in artificial organisms such as buildings and cities); (b) order is expressed by wholeness, that is, by an intensely intertwined system in which nothing is superfluous and everything is related to everything else in a meaningful way; (c) there is a sense of order, that is, wholeness operates as an attractor to which new parts and relationships tend to; and (d) wholeness can be broken down into particular properties, making the presence and intensity of each one in an organism result in more or less order. This is not essentially different from his previous theories, particularly from pattern language (Alexander et al., 1977), which too proposes immanent qualities to which objects and parts of objects should tend, except for here in HS things are much more far-reaching, in the sense that the focus is not on isolated objects but nature in general, that is, large natural as well as artificial systems.

Alexander's everlasting search for patterns (Alexander, 2002, 2009) acquires an expanded meaning, involving urban/architectural forms of operators (very many of them) trying to simultaneously understand existing underlying patterns in existing organisms, and acting towards generating their own order within such complex, uncontrolled environment—then, opposed to the Oregon experiment, where the whole group of agents is known and present at all interactions. Patterns would work like attractors, harmonious states to which forms should converge and, as in artificial systems forms do not evolve by themselves, human agents would seek for. However, patterns neither are unique nor individual's paths toward them are free from interferences, misunderstandings, or second thoughts, nevertheless, agents are many, independent, not present at all interactions, and unknown to each other. In this sense, an HS behaviour theory that does not consider human relative free will, learning skills and, above all, a context of thousands of independent and simultaneous actions interfering with each other, leading to unexpected results is missing something.

Our understanding, to this moment, is that:

1. There is an unresolved inconsistency between two important components of the theory, one dealing with patterns, taken as immanent qualities of form, and the other dealing with HS itself, the latter suggesting some sort of reasoning. In natural organisms, HS is, in fact, a passive process of form production led by cosmic forces, volcanic activity, tectonic plaques' movement, gravity, Brownian interaction, cell partition, and so on. In artificial organisms HS involves thinking, comparing, expecting, learning, mistaking, and evaluating, all of that practised by a great number of operators supposedly cooperating with each other (although not all with all) over space and time.

2. There is also an unresolved inconsistency between the very concept of wholeness, a sort of synthetic quality, obtained through a sufficient presence of virtuous patterns, and the HS process itself. Conscious actions of HS carried out by a specific operator could not only undermine similar actions taken by other operators on the same organism, destroying their carefully crafted patterns, but also failing to achieve its own objective by being undermined by others.

3. There is a lack of definition about what the actual 15 properties of wholeness are, as well as a complete lack of relationship between them and the HS process. Alexander believes that these properties (levels of scale, strong centres, boundaries, alternating repetition, positive space, good shape, local symmetries, deep interlock and ambiguity, contrast, gradients, roughness, echoes, the void, simplicity and inner calm, and not-separateness) are crucial in making the wholeness of a system. However, Seamon (2016, p. 61) argues that the 15 properties "may cast an incomplete understanding when one attempts to apply them to the larger-scale environment," mainly because of the largely localist nature of most of them (e.g., local symmetries, good shape, and contrast). According to Seamon, the global scale is largely unaddressed by the properties, although wholeness is claimed to be a global character of configurations. The comprehensive character of Alexander's concepts, which deals with all scales, from rooms and buildings to neighbourhoods and whole cities, hampers their clear understanding. While the 15 properties can be useful to analyse simple artefacts like architectural facades, as suggested by Salingaros (1997), they may not be adequate for analysing large and complex artefacts such as cities, given the multidimensional and wicked nature of planning and urban design.

There is no intention here to fill the gaps and link the dots between Alexander's high theories, however, we could not avoid thinking about what he came up with, and, following his advice, try to explore alternative approaches to the problem. We start with the relation between properties (patterns) and values (immanent qualities), which has been indeed discussed before (Alexander & Poyner, 1984; March, 1976; Rittel & Webber, 1973). The latter authors did make the case for the wicked nature of the urban realm, a planning landscape full of ill-defined problems, ambiguous goals and objectives, uncertainty, and unclear alternatives, suggesting a fragmented framework for pattern and value recognition. March (1976) goes in the same direction, arguing the immanent qualities and defending that values are fundamentally social, or socioeconomic, something that can change according to different circumstances, people involved, resources available, priorities, etc. Such vision seems more compatible with the idea of a complex organism being built over time by many agents acting independently from each other. Each individual HS action can be contradicted or undermined by others right on the same spot and at the same time. In this sense, everybody is looking for something (beauty, wholeness) but hardly getting exactly what they envisioned.

Alexander himself, together with Poyner, in their article "The Atoms of Environmental Structure," seems to offer a view over the conflicting nature of environment evolution. They say that:

> The environment requires a specific geometry only to resolve a conflict between two tendencies [tendencies is the word they use to express "needs"], and once a conflict between two tendencies is clearly stated, it would be possible to define the geometrical relation required to prevent the conflict. (Alexander & Poyner, 1984, p. 124)

Additionally, "the environment needs no geometrical organization beyond that which it gets from combination or relations so defined" (Alexander & Poyner, 1984, p. 124). In our own words, this is understood as, first, a conflict in the process, second, conflicts are prompted by two (or many) agents acting simultaneously on the same spot, generating conflict, third, agents act according to each own needs (meaning values) and fourth, urban form emerges from a proper articulation between two (or many) needs.

In order to progress, we have derived some assumptions, which follow:

1. Wholeness can be represented by centrality, which is a well-known property of urban systems and can be measured in different ways. Centrality, perhaps, is not enough to encompass Alexander's concept of wholeness, although it is certainly as close as one can get to it in qualitative and quantitative terms.

2. Wholeness properties are taken as needs, or tendencies, i.e., sociotechnical components of a city that are represented by physical devices as well as socioeconomic values and can develop an individual's centrality effects, as well as act in combinatorial ways to generate complex centralities.

3. Centrality descriptions are manifest in terms of configurations rather than geometries so that different scales of urban can be explored. Configurations are meant to be relations between two tendencies that can prevent conflict, not excluding eventual geometrical derivations.

Relying on the above assumptions, this article aims to propose a graph-based approach as an alternative to operationalize the HS process. The scale addressed is the neighbourhood or the whole city scale, namely the urban planning/design scale.

In the next section, we detail the framework suggested to describe, analyse, and evaluate each component of the urban spatial system in relation to the others. We explain the descriptive system adopted and the spatial differentiation measures used to compose the proposed framework. Then, we present some exploratory studies, which illustrate how the suggested approach could work in an empirical context. Finally, we discuss the main drawbacks, challenges, and potential for future development.

2. Proposed Methodological Framework

An outstanding aspect of Alexander's trajectory is the focus on the design as a question of the relationship between parts and wholes (Mehaffy, 2019; Seamon, 2016). The 15 properties were his last attempt to identify the precise mathematical structure of that relationship, in the sense that the properties are structural features which describe 15 kinds of relationships among centres. Centres are the primary entities of which wholeness structure is composed and can be a wall, a building, an open space, or an entire city. Indeed, Alexander could not find a mathematical language to represent this, so the key step in this debate is to define how to describe urban components and their relationship to each other and the whole.

In that regard, Alexander's ideas (2002, 2009) are very suggestive of a network approach, as already have been explored by some authors (Jiang, 2015, 2016). Here we propose a step further: a multilayer network approach (Aleta & Moreno, 2019; Kivelä et al., 2014; Nicosia et al., 2013). Bearing in mind the notion of centres (Alexander, 2002) as entities that represent some bit of geometry/space we attempt to schematize it into a graph language and describe their relationships by graph-based measures, like centrality measures.

Graph-based approaches are widely used in urban studies and provide formal representation and a mathematically manageable language to handle the urban components. There are several street network models within graph-based urban studies, from axial lines to road intersections (Marshall et al., 2018), but most of them focus on the representation solely of the street network, as a simple graph, while other components of the urban system such as built forms, land use, transport infrastructure and symbolic relationship receive less attention.

Some authors (Aleta & Moreno, 2019; Kivelä et al., 2014) suggest that simple graphs can be an obstacle to the representation of some phenomena because they focus on one type of relationship at a time. Nicosia et al. (2013) argue that a complex network is rarely isolated, and often some of its nodes could be part of several graphs at the same time. Thus, there is a growing interest among network scientists in a perspective of multilayer graphs, which tends to be a more realistic representation of complex systems, when considering multiple types of elements and relationships (Kivelä et al., 2014; Nicosia et al., 2013). Such a perspective is especially useful to handle urban systems since a city can be thought of as a large system composed of subsystems, which, in turn, are also composed of subsystems (Johnson, 2012). This refers to the vision presented, decades ago, by Alexander (2015) in "A City Is Not a Tree" (originally published in 1965) which emphasizes the overlapping subsystems observed in cities. In that sense, an approach based on multilayer graphs, in which vertices and edges represent elements and relationships of various types, opens the possibility of operationalizing the representation of the different networks and structures that overlap in the city.

Some authors have already explored a multilayered graph perspective to handle multimodal networks of transport (Gil, 2014) while others have explored some way of including built forms or land use in graph-based representations (Krafta, 1994, 1996; Krüger, 1979; Sevtsuk & Mekonnen, 2012) and cognitive structure

(Faria, 2010; Faria & Krafta, 2003, 2013). But there is still no unified framework to represent multiple kinds of urban features. Thus, the main advantage of the descriptive system suggested here is to provide a disaggregated description of the urban organism embracing several kinds of urban components, both qualitative and quantitative. Such a descriptive system can tackle multiple dimensions of urban design, such as physical attributes, activities, and conceptions (Montgomery, 1998).

The next sections detail the descriptive system and the spatial differentiation measures adopted in this article.

2.1. Description of the Urban Organism

We propose a descriptive system based on two key ideas: (a) Alexander centres as the nodes of a graph; and (b) the urban spatial system as a multi-layered graph. The first step for developing such an approach is to define what are the nodes (vertices) and the links (edges) of each layer in the graph. Then it must be defined how each layer becomes part of the graph, that is, which elements can be represented as vertices and what types of connections must be considered. This also involves thinking about how different layers interact with each other.

A general scheme of the urban system can be seen in Figure 1, with its systems and subsystems categorized into three groups: urban spatial structure, functional structure, and cognitive structure. The urban spatial or morphological structure refers to the physical–spatial dimension, encompassing elements such as public spaces, built forms and infrastructure, each of which consists of its own subsystems. The functional structure, characterized by the different networks of actors—individuals, groups, and institutions—concerns the socioeconomic dimension of the urban spatial system. It comprises a system of interactions, which is strongly influenced by elements geographically located and distributed in the urban space, such as land use, activities, densities, and transportation facilities. Lastly, the cognitive structure, which refers to the mental representation of the environment, consists of various elements that individuals or communities utilize to organize their mental image into meaningful information units.

A multidimensional framework was designed for this study, including specific representation strategies for each type of urban element, as shown in Table 1. Representation strategies refer to the rules or mechanisms for constructing the graph and can vary according to the adopted perspective. For instance, built forms can be viewed as individual objects represented by nodes added to the graph, or from a functional perspective where other representation strategies can be employed, such as adding land use information and weights to the vertices. The representation of the street network as a graph already has a vast literature (Marshall et al., 2018). Other elements, however, lack a greater definition of how to describe them through graphs. Therefore, the strategies outlined here help provide a unified framework for multi-layer graph representation.

For the functional dimension weighted graphs, directed graphs, and remote connections are the main strategies suggested. Weighted graphs mean assigning attributes to the nodes that give some loading effect to the graph so that it no longer deals only with spatial configuration. Such attributes can refer to urban densities or land value, for example, which are usually unevenly distributed in the urban system. It is also possible to use directed graphs, in other words, to specify pairs of complementary activities, such as supply and demand or origin and destination so that functional dependency relationships between pairs of specific nodes are characterized. Such representation strategy enables running specific graph analysis considering only selected pairs

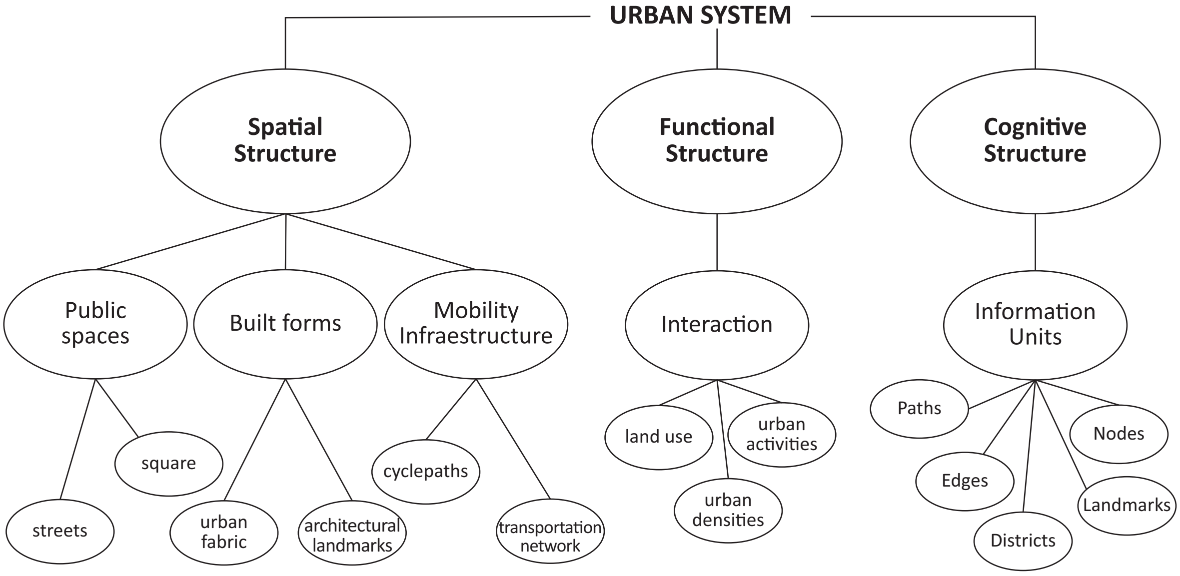


Figure 1. Examples of elements that comprise the urban system's representation.

Table 1. A multidimensional framework for multi-layer graph representation.

	Urban elements	Representation strategies
SPATIAL/MORPHOLOGICAL DIMENSION	Public spaces, street network; built forms; building typologies; parcels, mobility infrastructure	(a) Simple graphs (b) Multilayered graphs (c) Use of impedance values on edges
FUNCTIONAL DIMENSION	Land use; urban activities; urban densities; transport	(a) Weighted graphs (b) Directed graphs between complementary pairs of activities (c) Remote connections between elements
COGNITIVE DIMENSION	Public image; symbolic relationships	(a) Aggregation of elements (b) Distinction of elements (c) Remote connections between elements with some cognitive or symbolic relationship

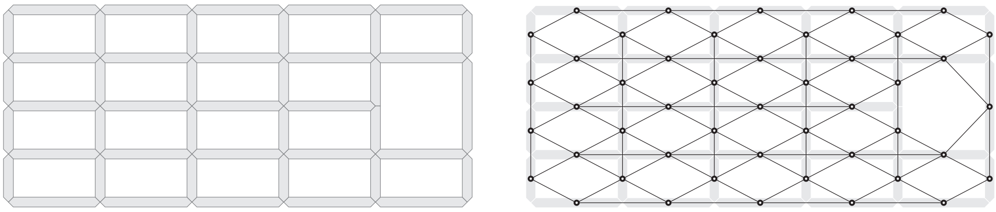
of nodes and not the "all-to-all" analysis (e.g., Krafta, 1996), as commonly used in the spatial configurational analysis. Using remote connections, which means not being by physical adjacency, is another strategy for representing functional aspects of cities, such as transportation. Remote connections between public transport stop, for example, can be characterized by edges that apply a shortcut effect to the graph (e.g., Gil, 2014). It can also be used to directly associate urban activities connections.

For the cognitive dimension, the representation must schematize the mental representations of the environment. Mental representation concerns the cognitive structure that each person or community has on the environment, in other words, it consists of environmental information cognitively structured in the human mind (Faria & Krafta, 2003). This type of description is practically unexplored as graph-based models, except for Faria and Krafta (2003, 2013) and Faria (2010). According to these authors, there are two main criteria used by individuals in the process of identifying environmental elements, such as Lynch's five elements. The first is information aggregation, which is detecting continuities in the environment to define useful grouping of information, such as districts and paths. The second is information distinction or segregation, which means recognizing "significantly different or strategically located information, in order to create environmental elements for orientation and reference, such as nodes and landmarks" (Faria & Krafta, 2003, p. 5). Based on these mechanisms, it is possible to define strategies to represent the information units, such as (a) aggregation of the vertices that compose it into a single vertex, (b) multiple connections between the vertices that make up the same information unit, and (c) insertion of a new vertex to represent a distinguishable element. Besides being selected or grouped, environmental elements tend to have their relationship altered and distance distorted in the process of environmental cognition, so using edges that characterize remote connections between elements with some symbolic relationship can also be a valuable representation strategy.

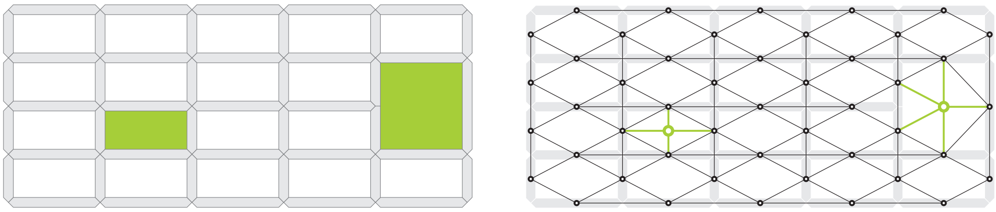
Figure 2 provides some examples of how each layer becomes part of the graph. Street networks, when taken from a purely spatial perspective, can be represented by simple graphs, and there is a plurality of approaches to network modelling (Marshall et al., 2018). In this article, we adopt a base graph representation where street segments are the vertices, and the junctions are the edges (Figure 2a). Marshall et al. (2018) refer to this as the "street-segment graph." This representation enables the description of not only the linear elements of the street network but any element of the urban system, regardless of its geometry. As a result, it is possible to encompass all the heterogeneity of the urban environment. Such disaggregated representation allows not only inserting new layers of elements more easily but also making flexible the inclusion of edges that correspond to different types of relations, without necessarily being by physical adjacency.

Several layers can be added to the base graph, as shown in Figure 2. This way, two or more layers can be combined in a unified representation scheme, always having the street network as the base graph. In the present study, parks and squares are represented as nodes connected to the adjacent streets (Figure 2b). Each route of public transportation is represented as a vertex, which does not physically exist, but represents the abstract idea of a public transport route. Such representation assumes that this route can take people directly to any stop point (Figure 2c). Residential use is assigned to the nodes by disaggregation of information from census data, while non-residential activities such as urban facilities, retail and urban equipment are assigned to the vertex corresponding to the adjacent street (Figure 2d). Information units of environmental cognition are represented by vertices connected to corresponding adjacent streets (Figure 2e). It is important to emphasize the possibilities of representation are not limited to these layers or these criteria.

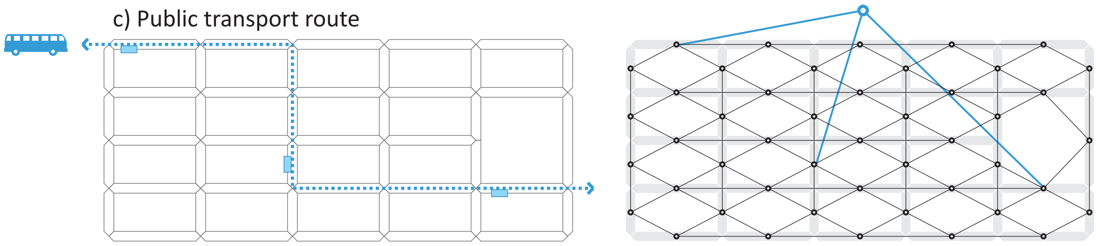
a) Street network



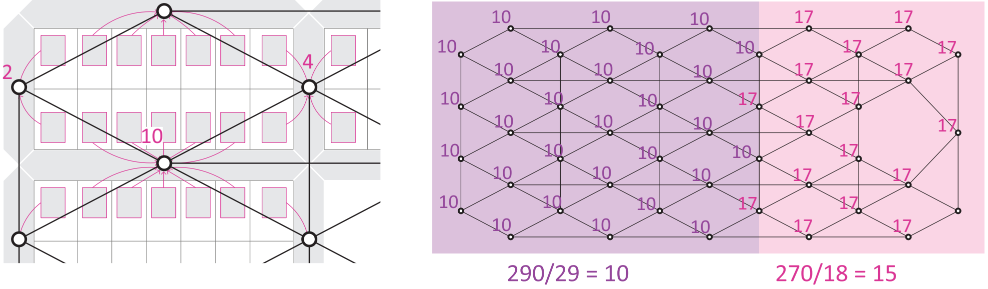
b) Parks, squares ou green spaces



c) Public transport route



d) Land Use



e) Information Units

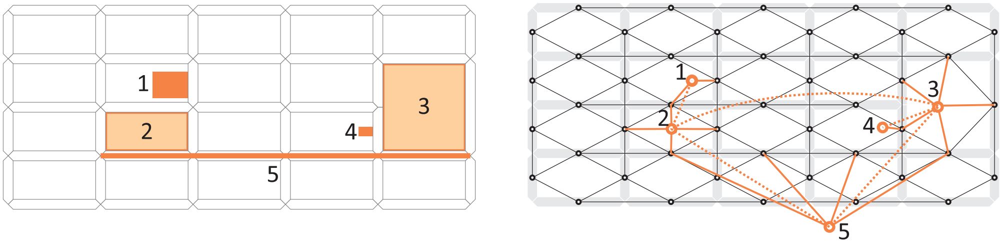


Figure 2. Examples of graph representation for each kind of urban component.

2.2. Spatial Differentiation Measures and Urban Properties

After outlining a representational framework, it is necessary to define methods of spatial differentiation for the graph's elements. Here we define a set of spatial differentiation measures proper to identify hierarchical relationships between entities, in other words, to describe the degree of wholeness, by measuring the effect of each component on all others. There are several graph-based algorithms proposed in the literature to depict network properties, firstly applied to social networks (Freeman, 1977) and afterwards to spatial networks (Crucitti et al., 2006; Hillier & Hanson, 1984; Krafta, 1994, 1996; Sevtsuk, 2010).

Graph-based measures capture the global characteristics of the configuration and allow for hierarchising the parts of the system, thus being useful to describe wholeness. Besides, network properties are mathematically manageable and can be related to urban properties, being useful to planning and urban design since several dimensions of interest in urban design are included.

According to Crucitti et al. (2006), a spatial analysis grounded on a set of different centrality measures rather than a single one increases the ability to characterize the city structure. For urban design purposes and aiming at a more holistic understanding, it seems logical to use a set of measures which highlight multiple kinds of hierarchies of the urban system. Combining different graph-based measures with different graph representations can lead to a set of several schemes to depict spatial differentiation, that can be associated with urban properties, like urban intensity (Krafta, 1994; Sevtsuk, 2010) or natural movement (Hillier et al., 1993).

Since the layers are independent of each other, many combinations are possible, always keeping the base layer of street networks which articulates the others. Through the insertion of different representation layers in the graph and attributes, it can reveal several urban properties, many of which have already been explored in literature. Since the layers are independent of each other, many combinations are possible, always keeping the base layer of street networks which articulates the others. Not necessarily all layers need to be involved in all measurements. Therefore, combining different graph-based measures with different graph representations can lead to a set of several schemes to grasp urban properties. The idea of having multiple representations of attributes and measurements of properties of the urban spatial structure meets the purpose of providing each operator (urban designer) with the possibility of associating properties of the urban spatial system with values and design objectives.

Several studies have found consistent correlations between centrality measures and empirical urban properties such as urban intensity (Krafta, 1994; Sevtsuk, 2010), co-presence patterns (Hillier & Hanson, 1984; Maciel & Zampieri, 2021), vehicle and pedestrian movement patterns (Hillier et al., 1993; Kirkley et al., 2018), land use patterns (Lima et al., 2017; Porta et al., 2009, 2012; Sevtsuk & Kalvo, 2018; Wang et al., 2011), land value patterns (Spinelli & Krafta, 1998), cognitive patterns (Faria, 2010) and stability of the urban structure (Kirkley et al., 2018; Strano et al., 2012), among others. Although still lacking further empirical evidence, the hierarchy captured by centrality measures reveals at least a latent potential of urban phenomena and socioeconomic behaviours. These indicatives, even provisionally, can be used as a proxy of urban properties to decision support in urban design.

In the next section, exploratory studies based on an empirical case are presented to illustrate the use of the proposed model, far from intending to exhaust all the possibilities based on the idea outlined here. The experiments carried out are not intended to validate the proposed model, however, they serve to discuss the potential and difficulties in operationalizing the design process with the procedures provided here, as well as to complement and detail its definition.

3. Experiments

The experiments presented here aim to deepen and complement some methodological aspects that seem particularly pertinent, such as the representation through multilayer graphs and the visualization of the results. Since there is no established method of doing this in the literature, these experiments precisely aim to explore possibilities of representation by examining the effects of the insertion of new layers in the graph. Each representation layer added to the graph produces effects, changing the hierarchy of the results. There are two important questions here: What kind of impacts are produced by inserting new layers, and how can they be visualized? Such explorations are made through an empirical case, the central region of the municipality of Lajeado (Brazil). In general terms, the study comprises a comparison of the insertion of different layers to the simple graph, which represents only the street network. We assume that other layers are always anchored to the street network, which works as a kind of base layer—indispensable—to the others. Then, the effects of the insertion of different layers in the graph are verified: parks and squares, bus lines, land use, and cognitive structure. By effect, we mean changes in the results of the different measures of spatial differentiation so that the elements gain or lose relevance in the ranking.

The tests carried out consist of the following steps: (a) provide descriptions through multilayer graphs; (b) process spatial differentiation measures; and (c) compare, through advanced visualization techniques, the insertion of different layers of representation, one by one, with the simple graph—only of the street network, observing how each layer deforms the results.

The multilayer graphs were constructed from empirical data (Figure 3), adopting the representation strategies outlined in the previous section, such as weighted graphs and remote connections. Closeness centrality and Freeman-Krafta centrality (F-K centrality) were used to analyse the network properties of the graphs, firstly applied to the simple graph, and then to the four multilayer graphs presented in Figure 3.

Closeness centrality (Crucitti et al., 2006; Freeman, 1979) is a distance-based measure since it illustrates the idea of accessibility, showing how close each location is to all other locations. This metric is defined as the inverse of the total distance required by a node to reach all the others.

F-K centrality (Krafta, 1994) is adapted from betweenness centrality (Freeman, 1977, 1979), which is one of the most used in urban network studies. Betweenness centrality measures the capability of a node to be in the path of the others. In other words, the nodes that are most often part of the shortest path between all the

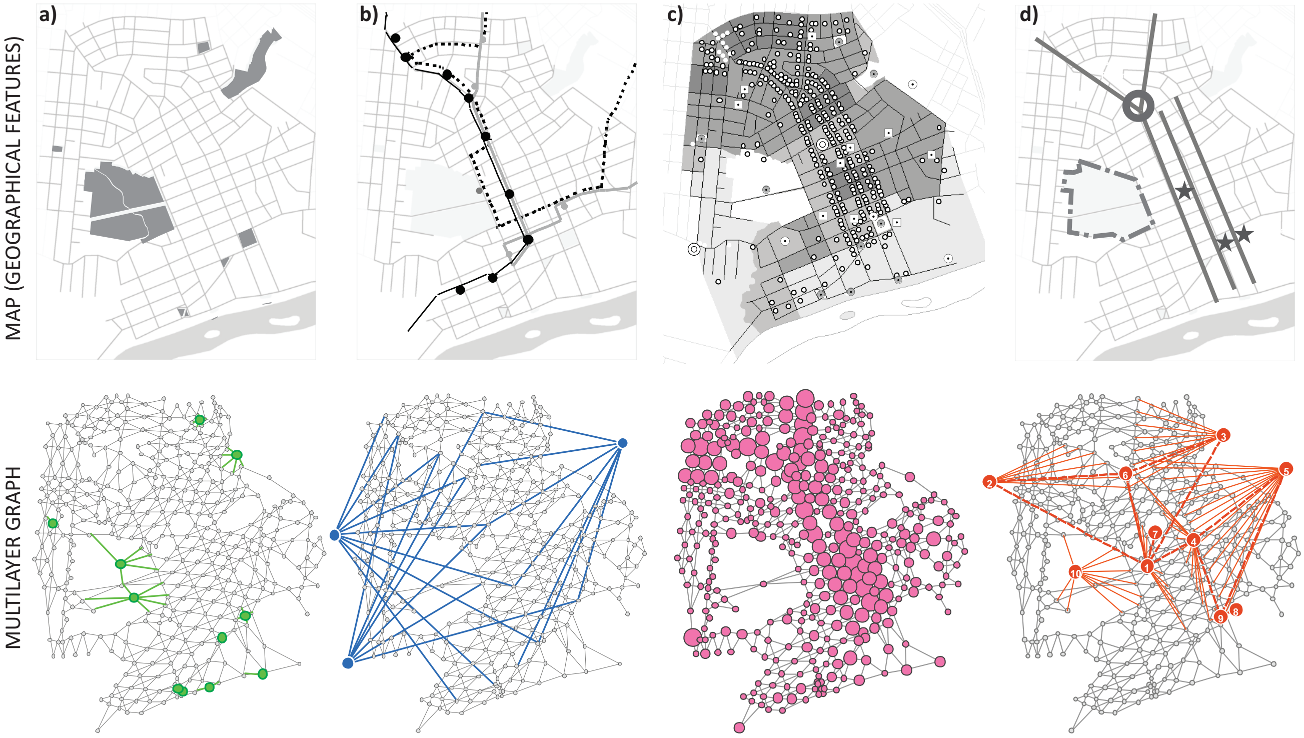


Figure 3. Map description and the corresponding multilayer graph of (a) parks and squares; (b) public transport routes; (c) land use—residential density, retail, and urban equipment; and (d) information units—paths, districts, nodes, and landmarks.

others are the most central in the graph. F-K centrality (Krafta, 1994) adopts the same calculation logic as betweenness centrality; however, it considers the distances between pairs of nodes and can compute node weights. F-K centrality was chosen because it describes more accurately the idea of urban spatial differentiation produced by geographic distances and uneven distribution of built forms or densities. Freeman's original measure computes the same value for all nodes that are in the path, no matter how long the path is, while F-K centrality considers that there is a tension between each pair of nodes, and this tension is dissipated along the path so that in pairs with more distant nodes the tension is diluted along the path. Since betweenness centrality was developed to analyse social networks, which are non-spatial, it does not consider these geographical and morphological aspects. Therefore, F-K centrality seems to be the most interesting for the analysis of urban networks.

All the measurements cited above analyse pairs of entities connected by a shortest path, that is, they are separated by a certain distance. In this study, distances are measured in the number of topological steps, since we use remote connections which are impossible to measure in metric distances.

To facilitate visualization and comparison, the results obtained for the graph nodes were converted into a raster surface using an interpolation technique. and the resulting surface was then normalized to a range from 0 to, as shown in Figure 4. One advantage of this visualization method, compared to visualization by discrete units directly on the graph nodes (as shown in Figure 4a) is the possibility to compare spatial patterns obtained from graphs of different sizes. These colour images clearly show hierarchical spatial patterns, revealing where the peaks of higher values are. They also show how the decay of these values occurs and how the values decay and distribute to the regions with lower values. With such images, it is easy to understand the role of each node in the global structure or, in other words, to understand its hierarchical position.

In this work, only visual comparisons were made. Thus, difference maps obtained through map algebra were also produced to facilitate the analysis and to help in comparing two raster surfaces (Figure 5). Difference maps were used to compare the results of each multilayer graph with the results of the simple graph, as they highlight areas with increased values (represented by warm colours) or decreased values (represented by cold colours) in comparison to the simple graph.

4. Results

Finally, Figure 5 summarizes the results obtained for closeness and F-K centrality measures in different combinations of layers in the graph in a raster surface ranging from 0 to 1 and its corresponding difference map. Through the difference maps, it is easy to see how the values obtained for the centrality measures change their hierarchy, with some regions showing an increase in

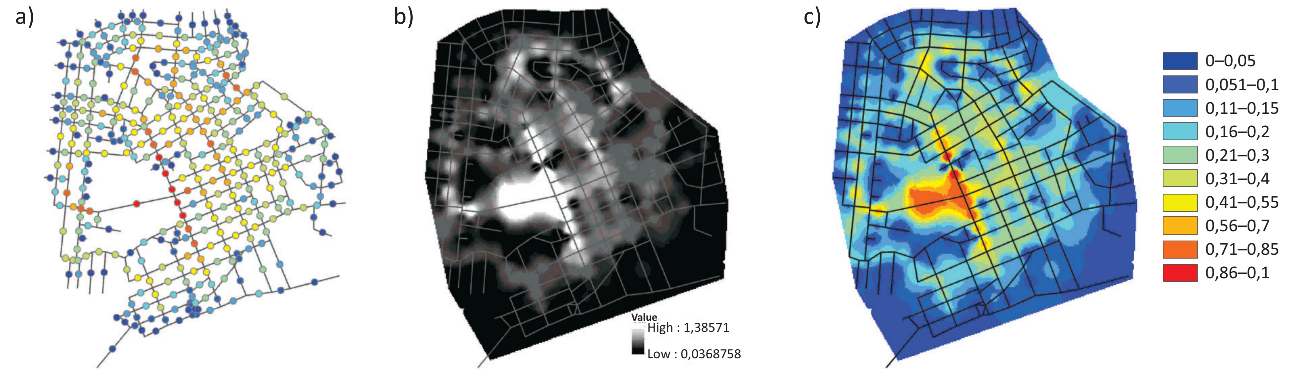


Figure 4. Geoprocessing process to obtain better visualization of results: (a) absolute results obtained for each node after running some centrality measure, (b) raster surface obtained by interpolation (inverse distance weighted technique) of the results from each node, and (c) normalized results from 0 to 1 range.

values and others showing a decrease. We can observe spatial pattern differences among the different centrality measures used and among the different representation layers.

Closeness centrality tends to concentrate higher values in the most central region of the graph, decreasing towards the edges, as it is a distance-based measure. Thus, representing only the street network does not offer great analytical potential. However, the insertion of other layers offers new perspectives for the use of this measure.

F-K centrality, as expected, presents results with an exponential statistical pattern, distinct from closeness centrality, and with quite distinct spatial patterns. In general, it can be said that it produces higher differences in the results than closeness centrality. Some layers produce more significant changes than others, depending on the representation strategy used. As we can see, the representation strategies that, in some way, generate a "shortcut" effect in the graph are the ones that produce the most significant impact on the results, as is the case of public transport and cognitive representation. Similar results can be found when adding weights to the nodes of the graph. In other words, such a representation strategy generates a "loading" effect, which deforms the hierarchy towards spaces with greater weights. Such results could be related to urban intensity. It is worth noting that closeness centrality does not consider the weights of the nodes so when we add land use weights to the graph, the result is the same as for the street network graph.

On the other hand, some representation strategies produce results with little difference compared to the simple graph, like just adding nodes in the graph. In the case of squares and parks, for example, the impact on the results depends on the position in the graph—more central or more peripheral—and the number of connections to the inserted node. In the case of the experiments performed here, only the biggest park of the study area led to great changes in the hierarchy.

The exploratory studies presented here are a small sample of the possibilities that can be explored through a multilayer network perspective. The same methodology used in the experiments to compare different graphs with different layers can be used in a design context, to compare and evaluate design hypotheses, verifying changes in the urban structure.

5. Discussion

The methodological framework proposed in this article has been designed to handle Alexander's HS computations and it can be thought of as an HS model of urban design. By assuming that design hypotheses contain values that are not always universal, we reject Alexander's vision of intrinsic quality of form grounded in the 15 properties. The harmony sought in a design context depends on the objectives and wills of the different agents. In other words, it varies according to the situation.

HS is reinterpreted here as a search for a design intention accomplishment. Therefore, we propose that the harmony sought in urban design could be given by a set of spatial differentiation measures, i.e., centrality measures that represent desirable urban properties. The disaggregated description of the urban system in the form of multilayer graph nodes could be a possible way to represent Alexander's centres, i.e., the entities of which the wholeness structure is composed. The graph-based approach outlined here and illustrated through exploratory studies provides a framework to deal with the whole and the role of the parts. Thus, it can be one possible way to answer Alexander's claims, at least for large-scale environments, since it enables description, analysis, and performance evaluation of each component, as well as the whole system, both through quantitative and qualitative representation.

The main contribution of the present work is precisely the discussion of Alexander's ideas focused on the scale of planning and urban design. The proposed framework seems to be a reasonable way to operationalize the HS process in a design context since it allows the depiction of various global patterns related to different aspects of urban design.

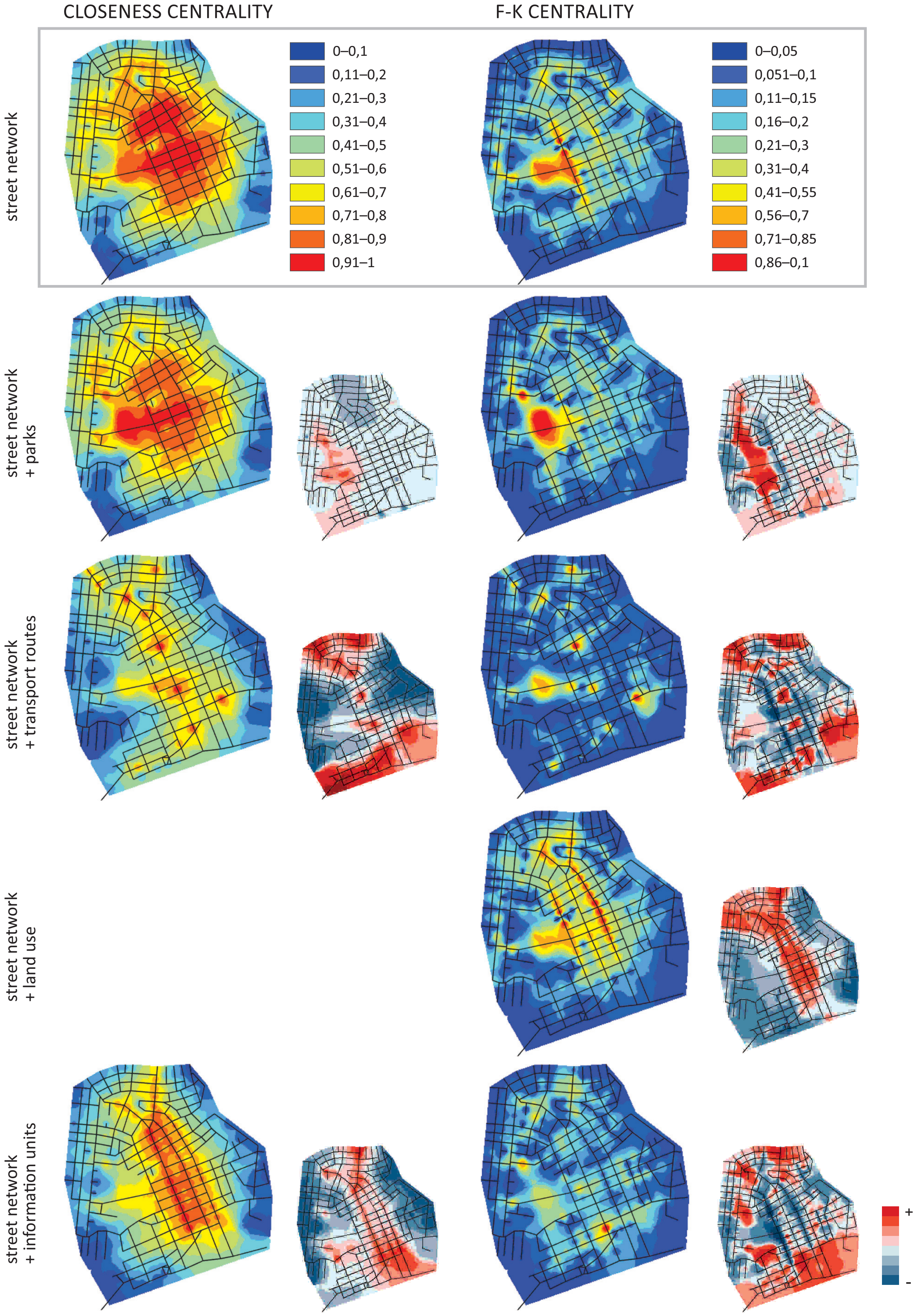


Figure 5. Results for closeness and F-K centrality measures in different combinations of layers in the graph and its corresponding difference map.

Given the number of possible measures that could be used, it would be up to the model operator (urban designer) to choose which variables would best represent the values sought in the project, in other words, the harmony sought in the project. Thus, the proposed model can be calibrated, through the insertion of different representation layers in the graph and attributes, allowing the operator to assign a relative value to any of its elements so that the concept of harmony fits the intended values and principles.

Mainstream urban configurational models focus on the description and analysis of the public spaces or street network. However, as seen throughout this article, there is still underexplored potential in representing other fundamental elements of urban design, such as built forms, urban infrastructure, cognitive aspects, and many others. The possibilities of representation and measuring are not restricted to those suggested here. In this sense, this work provides a starting point for further studies. Other centrality measures could be used, as well as other representation strategies could be explored. For instance, transport routes could be represented in different ways, like the representation used by Gil (2014). The representation of the cognitive structure is particularly challenging, having in mind the high degree of uncertainty regarding this topic and the reduced number of works that use graph-based approaches to describe the information units. Thus, the representation of the subjective dimension should be elaborated in future research. For example, different representation criteria could be thought of for the cognitive structure, considering different social groups. Additionally, complementary representation strategies also can be thought such as the definition of radii for processing measurements, and the definition of impedance values for the edges, which favour or disfavour certain paths, such as distance, travel time, road hierarchy, or slopes.

One of the main challenges for using a model with the proposed characteristics is translating requirements that are important to urban design to the possibilities of graph-based analysis results. In other words, more empirical works are needed to verify the correlation between graph-based measures and real phenomena.

A limitation of the proposed approach, based on data and modelling, is the very dynamic character of the city, which means that the simulation of city changes, that is, the design hypothesis, will always be out of step with what the city is at that moment. At the very moment that a project proposal is being developed and discussed, several aspects of reality are already changing, given that the process of urban changes is continuous.

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Conflict of Interests

The authors declare no conflict of interests.

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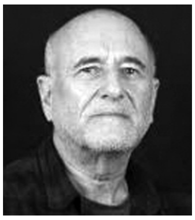

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